DOUBLE QUADRATIC BUCK CONVERTER

Franciéli L. de Sá, Caio V. B. Eiterer, Domingo Ruiz-Caballero, Samiram Mussa

Abstract - In this paper, a new non-insulated DC-DC converter called Double Quadratic Buck Converter is proposed. This new converter has the advantages of high gain ratio near half range of duty cycle compared to the conventional Buck Converter. The stress voltage over the switches is reduced to half of the value compared to the conventional Quadratic Buck Converter. The topological symmetry simplifies the theoretical analysis of the converter. A complete theoretical analysis is made. The converter operation stages with the respective waveforms are explained. Static gain, critical inductance and boundary conduction mode are equated in order to obtain the external behavior curve of the converter. Besides the static model will be presented in this paper the dynamic model of the converter and the control of input current and of output voltage.

Words - Keys – Buck Converter, Quadratic Buck Converter, Double Quadratic Buck Converter.

I. INTRODUCTION

Regarding the dc-dc power conversion, robustness, efficiency and low cost with topological simplicity are the main concern. In this perspective, Buck and derived converters are the first converter group to look after. Thus, new topologies derived from the Buck Converter have been investigated in several works.

New converter topologies were developed by combining existing converters. The association of the cascade converters, also called converters with more than one stage, originated quadratic converters. Conversion range can be extended significantly if conversion ratio has a quadratic dependence of static gain. Relating to the initial studies of dc-dc converters with cascaded association, the Cuk Converter with two stages, was presented in [1]. Later, in [2] was presented a cell switching responsible for generating a family of dc-dc converters, where originated the analysis to the development of the Quadratic Buck Converter. The study of the static model of the Quadratic Buck Converter was presented in [3]. The dynamic modeling and control of Quadratic Buck converter was developed in [4] and [5]. Among the many publications in the literature about the buck converter with high gain one can cite [6]. Relating yet, the multilevel converters with the traditional DC-DC converters are presented in [7] and [8].

Whereas the utilization factor of the switch of the Conventional Buck Converter is optimized for high values of duty cycle [9], namely, low conversion rate static, in applications requiring wide range of the static conversion the Double Quadratic Buck Converter becomes interesting. In this case, a wide range allows the static converter operate with duty cycle suitable for the same voltage applied to the Buck converter Conventional. In this context, this converter can be used in various applications related the use of batteries with a voltage lower than the voltage generating source, such as for example in systems generation photovoltaics [10], battery chargers for cell phones [11] and [12] switching power supplies [13], among others.

The Double Quadratic Buck Converter is characterized by average output voltage to be lower than input voltage, and the voltage intermediate capacitor also to be lower than the input voltage. Furthermore, it has high gain ratio compared to the conventional Buck Converter. In this case, the voltage at the switches is lower than the input voltage and half the voltage in the switch of the Quadratic Buck Converter existing in the literature.

In this paper the analysis of the operation stages, the ideal static gain curve plot, the study of the converter operating in continuous, critical and discontinuous operation mode, as well as the drawing of the external characteristic curve for the converter is presented. The simulation results presented were performed with the software "PSIM".

The dynamic model using the technique for state space, are presented in [14], [15], for the Buck Converter and [16] for the Quadratic Buck Converter, will be developed in this paper for the Double Quadratic Buck Converter. The PI controller and the simulation results, in Closed Loop, also will be presented.

II. CONVERTER TOPOLOGY

Likewise that the Buck Converter Conventional the Double Buck Quadratic Converter also has step-down voltage characteristic, that is, the output voltage is lower than the input voltage. However, by having high static gain has wide range in the output voltage.

In this structure, both the power supply and the intermediate capacitor will behave as voltage source. The load should behave as a current source and the current on the intermediate capacitor is given by the difference between the current across the inductor and the current in the switch. Because of its symmetrical topology, the lower components have the same behavior of the respective upper component. The Fig. 1 shows the topology of the Proposed Converter.

III. IDEAL STATIC GAIN OF THE CONVERTER

A. Continuous Conduction Mode

Analysis of the Operation Stages:
**First Stage: \((D.T_s)\)**

In this stage the switches \(S_1\) e \(S_2\) are turn on. The diodes \(D_2\) and \(D_6\) are inversely polarized, the current sources \(I_{L1}\) e \(I_{L2}\) start to deliver power to the output. The current \(i_{S1}\) is the sum of \(I_{L1}\) with \(I_{C1}\), and the current \(i_{D1}\) is null, Fig. 2 (a).

**Second Stage: \(((1-D).T_s)\)**

In this stage the switches \(S_1\) e \(S_2\) are turn off. The diodes \(D_3\) and \(D_6\) come into conduction, isolating the current source \(I_{L1}\) of the output and of the current source \(I_{L2}\). In this stage, the current \(i_{S1}\), \(i_{S2}\), \(i_{D2}\) and \(i_{D4}\) are nulls, \(i_{D3} = I_{L1}\) and \(i_{D3} = I_{L2}\), Fig. 2 (b).

The Figure 3 shows the waveforms to each operation stage.

To draw the curve of ideal static gain, it is considered the energy given by the source \(V_{in}\) in an operation period equal to (1).

\[
W_{in} = \left( \frac{V_C}{2} - \frac{V_{in}}{2} \right) I_{L1}.\Delta t_1
\]  

(1)

The energy received by the intermediate capacitors \(C_1\) and \(C_2\) in an operation period is given by (2).

\[
W_{C_{1,2}} = -\frac{V_C}{2} I_{L1} \Delta t_2
\]  

(2)

Considering an ideal system, in a period of operation all energy given by the source \(V_{in}\) is received by the intermediate capacitors \(C_1\) and \(C_2\). Thus, from equating (1) e (2) we obtain the Equation of Ideal Static Gain for the first part of the converter, as shown in (3).

\[
\frac{V_C}{V_{in}} = D
\]  

(3)

The same analysis is performed for the second part of the converter, considering \(V_C\) as the source of input and \(I_{L2}\) as the constant current source.

Using the principle of superposition, analyzing the first and the second stages of operation, is obtained the static gain optimal total of the Double Quadratic Buck Converter proposed depending on the output voltage by the input voltage, given in (4).

\[
\frac{V_0}{V_{in}} = D^2
\]  

(4)

The Fig. 4 shows the curve of the static gain as a function of duty cycle for the Double Quadratic Buck Converter Proposed, for purposes of comparison is also presented to the static gain curve Buck Converter Conventional.

![Double Quadratic Buck Converter CC-CC Proposed](image1)

**First Stage: \((D.T_s)\)**

![Operation Stages of Converter in Conduction Continuous Mode with a single command of the switches \(S_1\) e \(S_2\): a) First Stage; b) Second Stage.](image2)

![Waveforms Double Quadratic Buck Converter Proposed operating in continuous conduction mode.](image3)

![Ideal static gain of Double Quadratic Buck Converter proposed compared to the Ideal Static Gain of Conventional Buck Converter.](image4)
Based on Equation (3), the voltage on the switch S1 is given by the voltage source in the upper input \( V_{in} \) added to the voltage on the intermediate capacitor \( C \) \( (V_C/2) \). Thus we have:

\[
V_{S1} = \frac{V_{in}}{2} + \frac{V_C}{2} \quad (5)
\]

Considering the total input voltage \( V_{in} \) and also substituting \( V_C \) as a function of \( V_{in} \), the voltage on the switch \( S_1 \) is obtained:

\[
V_{S1} = \frac{3}{4} V_{in} \quad (6)
\]

The Equation (6) shows that the stress on the switch \( S_1 \) is small when compared to the Quadratic Buck Converter [2], and less than the total voltage at the input \( (V_{in}) \). The voltage on the switch \( S_2 \) is obtained similarly.

**B. Critical Conduction Mode**

The operation stages for the critical conduction mode are the same described for the continuous conduction mode. What distinguishes these two modes operation is the fact that the current in the inductors having minimum value \( I_{min} \) equal to zero. Thus, during the first stage of operation, the current in the inductors \( L_1 \) and \( L_2 \) are initially zero and vanish again exactly at the end of the period of operation of the converter.

The converter in critical conduction mode represented by the waveform shown in Fig. 5, with the respective time intervals of conduction of switches corresponding to each stage.

\[
\Delta I_{L1} = I_{L1,\text{max}} = \frac{V_{in}/2}{L_1} \cdot D. (1 - D) \quad (7)
\]

\[
\Delta I_{L2} = I_{L2,\text{max}} = \frac{V_C/2}{L_2} \cdot D. (1 - D) \quad (8)
\]

Given the minimum and maximum values of the input current \( I_{L1,\text{max}} \) and \( I_{L1,\text{min}} \) as a function of the current intermediate capacitor for continuous conduction mode, the critical inductance is found canceling out the current \( I_{L1,\text{min}} \).

\[
I_C = \frac{I_{L1,\text{max}}}{2} \quad (9)
\]

Substituting Equations (7) and (3) obtains (10). Thus the inductance \( L_1 \) critical is given by:

\[
L_{1,CR} = \frac{V_{in}/2}{2.\Delta D. I_{C}} D. (1 - D) \quad (10)
\]

Repeating the same analysis for the inductor \( L_2 \), the maximum and minimum values of input current \( I_{L2,\text{max}} \) and \( I_{L2,\text{min}} \) as a function of current output capacitor for continuous conduction mode, the critical inductance is found canceling out the current \( I_{L2,\text{min}} \).

\[
L_{2,CR} = \frac{V_C/2}{2.\Delta D. I_{0}} D. (1 - D) \quad (11)
\]

**C. Discontinuous Conduction Mode**

The operation stages for the discontinuous conduction mode are described below: The first and the second operation stages are identical to the continuous conduction mode, therefore will not be described again in this section. As remembered that \( D_1, T_3 \) is the conduction time for the 2nd stage operation.

Recalling that the duty cycle complementary in discontinuous conduction is given by \( 1 - D = D_1 + D_2 + D_3 \), according to the respective operation stages.

**Third Stage:** \((D_2, T_2)\)

In this stage all the energy stored in \( L_2 \) was transferred to the load. With this, the diode \( D_3 \) blocks and output capacitor \( C_0 \) feeds the load. The inductor \( L_1 \) continues to provide energy to the intermediate capacitor \( C_1 \).

**Fourth Stage:** \((D_1, T_1)\)

At this stage all the energy stored in \( L_1 \) and \( L_2 \) are transferred. Thus, the diode \( D_1 \) blocks and capacitor \( C_0 \) feeds the load.

The Figs. 6 and 7 show the stages of operation and the waveforms of the converter, respectively.

For the analysis of the static gain of the converter in discontinuous conduction mode, consider the inductor voltage \( L_1 \) for the 2nd operation stage in absolute value \( V_{L1} = V_C \) given by (12):

\[
V_C = \frac{L_{1,\text{Dis}}}{2} \cdot D_1 \quad (12)
\]

Observing Fig. 7, can get (13), for the 1st part of the analysis.

\[
I_C - \Delta I_{input} = \frac{I_{L1,\text{max}}}{2} \cdot D_1 \quad (13)
\]
Fig. 6. Operation Stages in Discontinuous Conduction Mode: a) 1st Stage; b) 2nd Stage; c) 3rd Stage; d) 4th Stage.

Isolating $I_{L_{1,\text{max}}}$ of (13) and equating to (12), may find (14) concerning the 1st part of the static gain of the ideal converter operating in discontinuous conduction mode:

$$\frac{V_C}{V_{in}}/2 = 1 - \frac{(V_C/2).D_1^2}{2.I_C.L_1,\text{Dis} \cdot f_s}$$  \hspace{0.4cm} (14)

Repeating the analysis for the $L_1$ inducing $L_2$, we obtain (15), referring to the 2nd part of the gain equation:

$$\frac{V_C/2}{V_{in}} = 1 - \frac{(V_0/2).D_1^2}{2.I_0.L_2,\text{Dis} \cdot f_s}$$  \hspace{0.4cm} (15)

To obtain the ideal total static gain converter operating in discontinuous conduction mode uses the principle of superposition (14) and (15), to give (16).

$$\frac{V_0}{V_{in}} = \left(1 - \frac{(V_0/2).D_1^2}{2.I_0.L_1,\text{Dis} \cdot f_s} \right)^2$$  \hspace{0.4cm} (16)

It is noted in (16), that the duty cycle $D$ should be able to compensate for both variations in the input voltage $V_{in}$, as the load variations $I_0$.

D. External Characteristic

Analyzing the equations of the static gain in Continuous Conduction mode (4), in discontinuous conduction mode (16), and making $a = V_0/V_{in}$ and $\gamma = 2.I_0.L_1,\text{Dis} \cdot f_s/V_{in}$, we have the equations of the static gain to continuous and discontinuous conduction mode in simplified form.

Making the necessary replacements, one obtains (17) representing the boundary between continuous conduction mode and discontinuous conduction mode. The Fig. 8 shows the curve that represents the external characteristic of the converter.

$$\gamma = \pm \sqrt{a} - a$$  \hspace{0.4cm} (17)

Fig. 7. Waveforms Double Quadratic Buck Converter Proposed operating in discontinuous conduction mode.

Fig. 8. External characteristic of the Proposed Converter.
The system can be represented by equations of input and output as shown in Equation 2. The classic control is used on systems that require only one input and one output. For systems that require multiple inputs and outputs, it is used to the state space modeling, obtaining models more accurate and that faithfully represent the system. The system can be represented by equations of input and output, as shown in Equations 18 and 19:

\[ K \frac{d}{dt} X(t) = A \cdot X(t) + B \cdot U(t) \]  
\[ Y(t) = C \cdot X(t) + E \cdot U(t) \]

**V. DYNAMIC MODELING AND CONVERTER CONTROL**

In this section will be presented the dynamic model using the technique of state space, the control the peak current in response to the load step and the control the output voltage to the proposed converter.

The classic control is used on systems that require only one input and one output. For systems that require multiple inputs and outputs, it is used to the state space modeling, obtaining models more accurate and that faithfully represent the system. The system can be represented by equations of input and output, as shown in Equations 18 and 19:

\[ K \frac{d}{dt} X(t) = A \cdot X(t) + B \cdot U(t) \]  
\[ Y(t) = C \cdot X(t) + E \cdot U(t) \]

For dynamic modeling considers the operation stages in continuous conduction mode:

**First Stage: \((D \cdot T_s)\)** From the analysis of Figure 2 (a), one can obtain the state matrices which determine the voltage in the capacitor and the inductor current, as shown in Equation 20:

\[ [i(t)] = \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ i_{L3}(t) \\ i_{C1}(t) \\ i_{C2}(t) \\ i_{C0}(t) \end{bmatrix} \]

\[ [V(t)] = \begin{bmatrix} V_{L1}(t) \\ V_{L2}(t) \\ V_{L3}(t) \\ V_{C1}(t) \\ V_{C2}(t) \\ V_{C0}(t) \end{bmatrix} \]

\[ [V_{in}(t)] = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

**Second Stage: \(((1-D) \cdot T_s)\)** The circuit illustrated in Figure 2 (b) is shown in Equation 21:

\[ [i(t)] = \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \\ i_{L3}(t) \\ i_{C1}(t) \\ i_{C2}(t) \\ i_{C0}(t) \end{bmatrix} \]

\[ [V(t)] = \begin{bmatrix} V_{L1}(t) \\ V_{L2}(t) \\ V_{L3}(t) \\ V_{C1}(t) \\ V_{C2}(t) \\ V_{C0}(t) \end{bmatrix} \]

\[ [V_{in}(t)] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]
The next step will be to get the equations that determine the average model for the two operation stages of the converter. The matrix "A" average is given by:

$$A = d.A_1 + d^* . A_2$$  \hspace{1cm} (22)

Similarly, one can find the values of the matrices B, C and E. With the values of the DC components, one can define the model AC small signals:

$$K \frac{d}{dt} \hat{x} = A. \hat{x}(t) + B. \hat{u}(t) + \{(A_1 - A_2).X + (B_1 - B_2).U\}. \hat{d}(t)$$  \hspace{1cm} (23)

$$y(t) = C. \hat{x}(t) + E. \hat{u}(t) + \{(C_1 - C_2).X + (E_1 - E_2).U\}. \hat{d}(t)$$  \hspace{1cm} (24)

where: $\hat{x}(t)$, $\hat{u}(t)$, $\hat{y}(t)$, $e \hat{d}(t)$, are small variations on the operating point.

With the matrices obtained for the first and second operation stages, as well as the equations that define the system in state space, one can obtain the transfer functions of the circuit, using the MATLAB software.

To prove the model in state space the Fig. 11 shows the Bode diagram of the transfer function of the output current as a function of duty cycle $G_{id}(s) = \frac{I_{d1}(s)}{d(s)}$ in comparison with the circuit obtained by the tool ACSweep PSIM software.

The Bode diagram of the transfer function of the output voltage by duty cycle $G_{vd}(s) = \frac{V_{d}(s)}{d(s)}$ is shown in Fig. 12 and compared with the circuit using the ACSweep.

Wishing that the inverter is able to reject variation in output voltage and frequency peaks in moments where there are load variation, the controllers for current and voltage loops were projected.

To control the inner current loop, the linear controller used is the proportional integral (PI) + pole seeking null error in steady state. The block diagram representing the internal current loop is shown in Fig. 13. The frequency of zero crossing designed was 5 kHz and the phase margin of 72 degrees. The equation 25, shows the projected current compensator:

$$C_i(s) = k_i . \frac{s + z_i}{s.(s + p_i)}$$  \hspace{1cm} (25)

To the control of the Converter a external voltage loop is included to change the amplitude of the reference current according to the load and thereby regulate the output voltage. For this, the voltage loop must have a high gain DC. However, it should not have a high bandwidth not to distort the current reference. A compensator proportional integral (PI) + pole can also be used for control the voltage loop, getting signal with zero steady state error. The zero of the controller was set at the pole plant. The second pole of the controller was added above the frequency zero crossing to minimize noise in the control loop. The Fig. 13 illustrates the control the outer loop of the voltage. The Equation 26, shows the voltage controller projected:
\[ C_v(s) = k_v \frac{s + z_v}{s(s + p_v)} \]  \hspace{1cm} (26)

The simulation results using the control internal current loop and external voltage loop are shown in Figs. 14, 15 and 16. Comparing with the control projected, the converter operating in open loop and just with control of the output voltage loop are also presented. Thus it can be seen that the overshoot and settling time are smaller when the current control and the voltage control are used. Besides the evidence through simulation results more accurate when applied controllers for current and voltage, this type of controller becomes interesting in controlling associated converters in parallel, as shown in [17] or still in control of DC motors, [18].

Considering other cases to be controlled, such as in cases where there is a difference in duty cycle control of the switches, in this situation, the neutral current would not be zero and the voltages and currents in the components would become unbalanced. An example of this unbalance would be the photovoltaic panels coupled to each input of the converter supplying different points of maximum power.

### VI. CONCLUSION

The study of a new Converter non isolated DC-DC step down Double Quadratic Buck Converter was presented in this paper. The topology of the converter, the stages of operation and the lifting of the external characteristic curve was presented. Simulation results are presented for continuous conduction mode and show the low value in the output voltage.
over the input value, proving the high conversion rate of the converter. Furthermore, because of its symmetry the voltage stress in the switches in $S_1$ and $S_2$ are identical. Notwithstanding, since the inductors are important elements in determining the driving mode of the converter, the currents thereof are presented for the three situations, continuous and discontinuous critical, confirming the theoretical analysis. The dynamic model and the simulation results of the converter control were presented. Thus it will be possible to use this new converter in various applications such as the generation of photovoltaic energy, microgrids, electric vehicles and others.

REFERENCES


